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Random C^*-ternary algebras and application

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Abstract

In this paper, we introduce the concept of random C^*-ternary algebras and consider some properties of them. As an application we approximate a random C^*-ternary algebra homomorphism in these spaces.

MSC: Primary 39B52; 17A40; secondary 46B03

Keywords: random C^*-ternary algebra; C^*-ternary algebra homomorphism; random complex Banach spaces

1 Introduction

Ternary algebraic operations were considered in the 19th century by several mathematicians, such as Cayley [1], who introduced the notion of cubic matrix which, in turn, was generalized by Kapranov *et al.* [2]. The simplest example of such a non-trivial ternary operation is given by the following composition rule:

$$\{a,b,c\}_{ijk} = \sum_{1\le l,m,n\le N} a_{nil}b_{ljm}c_{mkn}$$

for each $i,j,k = 1,2,\ldots,N$.

Ternary structures and their generalization, the so-called n-ary structures, raise certain hopes in view of their applications in physics. Some significant applications are as follows (see [3, 4]):

(1) The algebra of nonions generated by two matrices

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega^2 & 0 & 0 \end{pmatrix},$$

where $\omega = e^{\frac{2\pi i}{3}}$, was introduced by Sylvester as a ternary analog of Hamilton's quaternions (see [5]).

(2) The quark model inspired a particular brand of ternary algebraic systems. The so-called Nambu mechanics is based on such structures (see [6]).

2 Random C^*-ternary algebra

In the section, we adopt the usual terminology, notations and conventions of the theory of random C^*-ternary algebra.

Throughout this paper, Δ^+ is the space of distribution functions, that is, the space of all mappings $F : \mathbf{R} \cup \{-\infty, \infty\} \to [0,1]$ such that F is left-continuous and non-decreasing on $\mathbf{R}$, $F(0) = 0$, and $F(+\infty) = 1$. D^+ is a subset of Δ^+ consisting of all functions $F \in \Delta^+$ for which $l^- F(+\infty) = 1$, where $l^- f(x)$ denotes the left limit of the function f at the point x, that is, $l^- f(x) = \lim_{t\to x^-} f(t)$. The space Δ^+ is partially ordered by the usual point-wise ordering of functions, *i.e.*, $F \leq G$ if and only if $F(t) \leq G(t)$ for all t in $\mathbf{R}$. For example an element for Δ^+ is the distribution function ε_a given by $\varepsilon_a(t) = 0$, if $t \leq a$ and 1 if $t > a$.

The maximal element for Δ^+ in this order is the distribution function ε_0 (see [7–9]).

Definition 2.1 ([8]) A mapping $T : [0,1] \times [0,1] \to [0,1]$ is called a *continuous triangular norm* (briefly, a continuous t-norm) if T satisfies the following conditions:

(a) T is commutative and associative;
(b) T is continuous;
(c) $T(a,1) = a$ for all $a \in [0,1]$;
(d) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1]$.

Typical examples of continuous t-norms are $T_P(a,b) = ab$, $T_M(a,b) = \min(a,b)$, and $T_L(a,b) = \max(a+b-1,0)$ (the Lukasiewicz t-norm).

Definition 2.2 ([9]) A *random normed space* (briefly, RN-space) is a triple (X,μ,T), where X is a vector space, T is a continuous t-norm and μ is a mapping from X into D^+ such that the following conditions hold:

(RN1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;
(RN2) $\mu_{\alpha x}(t) = \mu_x(\frac{t}{|\alpha|})$ for all $x \in X$, $\alpha \neq 0$;
(RN3) $\mu_{x+y}(t+s) \geq T(\mu_x(t), \mu_y(s))$ for all $x,y \in X$ and $t,s \geq 0$.

Every normed space $(X, \|\cdot\|)$ defines a random normed space (X,μ,T_M), where

$$\mu_x(t) = \frac{t}{t + \|x\|}$$

for all $t > 0$, and T_M is the minimum t-norm. This space is called the *induced random normed space*.

Definition 2.3 ([10]) A *random normed algebra* (X,μ,T,T') is a random normed space (X,μ,T) with algebraic structure such that

(RN4) $\mu_{xy}(ts) \geq T'(\mu_x(t), \mu_y(s))$ for all $x,y \in X$ and $t,s > 0$, in which T' is a continuous t-norm.

Every normed algebra $(X, \|\cdot\|)$ defines a random normed algebra (X,μ,T_M,T_P), where

$$\mu_x(t) = \frac{t}{t + \|x\|}$$

for all $t > 0$ if and only if

$$\|xy\| \leq \|x\|\|y\| + s\|y\| + t\|x\|$$

for all $x,y \in X$ and $t,s > 0$. This space is called the *induced random normed algebra*. For more properties and examples of the theory of random normed spaces, we refer to [11–27].

Definition 2.4 Let $(\mathcal{U}, \mu, T, T')$ be a random Banach algebra. Then an *involution* on $\mathcal{U}$ is a mapping $u \to u^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies the following conditions:

(1) $u^{**} = u$ for $u \in \mathcal{U}$;

(2) $(\alpha u + \beta v)^* = \overline{\alpha} u^* + \overline{\beta} v^*$;

(3) $(uv)^* = v^* u^*$ for $u, v \in \mathcal{U}$.

If, in addition, $\mu_{u^*u}(ts) = T'(\mu_u(t), \mu_u(s))$ for all $u \in \mathcal{U}$ and $t, s > 0$, then $\mathcal{U}$ is a random C^*-algebra.

Following the terminology of [28], a non-empty set G with a ternary operation $[\cdot,\cdot,\cdot] : G \times G \times G \to G$ is called a *ternary groupoid* and is denoted by $(G, [\cdot,\cdot,\cdot])$. The ternary groupoid $(G, [\cdot,\cdot,\cdot])$ is called *commutative* if $[x_1, x_2, x_3] = [x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}]$ for all $x_1, x_2, x_3 \in G$ and all permutations σ of $\{1, 2, 3\}$.

If a binary operation $\circ$ is defined on G such that $[x, y, z] = (x \circ y) \circ z$ for all $x, y, z \in G$, then we say that $[\cdot,\cdot,\cdot]$ is derived from $\circ$. We say that $(G, [\cdot,\cdot,\cdot])$ is a *ternary semigroup* if the operation $[\cdot,\cdot,\cdot]$ is *associative*, *i.e.*, if

$$\bigl[[x, y, z], u, v\bigr] = \bigl[x, [y, z, u], v\bigr] = \bigl[x, y, [z, u, v]\bigr]$$

for all $x, y, z, u, v \in G$ (see [29]).

A *random C^*-ternary algebra* is a random complex Banach space A, equipped with a ternary product $(x, y, z) \mapsto [x, y, z]$ of A^3 into A, which are $\mathbf{C}$-linear in the outer variables, conjugate $\mathbf{C}$-linear in the middle variable, associative in the sense that

$$\bigl[x, y, [z, w, v]\bigr] = \bigl[x, [w, z, y], v\bigr] = \bigl[[x, y, z], w, v\bigr],$$

and satisfies

$$\mu_{[x,y,z]}(tsp) \geq T\bigl(\mu_x(t), \mu_y(s), \mu_z(p)\bigr)$$

and

$$\mu_{[x,x,x]}\bigl(t^3\bigr) \geq T\bigl(\mu_x(t), \mu_x(t), \mu_x(t)\bigr)$$

(see [28, 30]).

Every random left Hilbert C^*-module is a random C^*-ternary algebra via the ternary product $[x, y, z] := \langle x, y \rangle z$.

If a random C^*-ternary algebra $(A, [\cdot,\cdot,\cdot])$ has the identity, *i.e.*, an element $e \in A$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in A$, then it is routine to verify that A, endowed with $x \circ y := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^*-algebra. Conversely, if $(A, \circ)$ is a unital C^*-algebra, then $[x, y, z] := x \circ y^* \circ z$ makes A into a C^*-ternary algebra.

A $\mathbf{C}$-linear mapping $H : A \to B$ is called a *C^*-ternary algebra homomorphism* if

$$H\bigl([x, y, z]\bigr) = \bigl[H(x), H(y), H(z)\bigr]$$

for all $x, y, z \in A$. If, in addition, the mapping H is bijective, then the mapping $H : A \to B$ is called a *C^*-ternary algebra isomorphism*. A $\mathbf{C}$-linear mapping $\delta : A \to A$ is called a C^*-

ternary algebra derivation if

$$\delta([x,y,z]) = [\delta(x),y,z] + [x,\delta(y),z] + [x,y,\delta(z)]$$

for all $x,y,z \in A$ (see [28, 31]).

There are some applications, although still hypothetical, in the fractional quantum Hall effect, the non-standard statistics, supersymmetric theory, and the Yang-Baxter equation (*cf.* [5, 32, 33]).

Throughout this paper, assume that p, d are non-negative integers with $p+d \geq 3$ and A, B are random C^*-ternary algebras.

Definition 2.5 Let (X,μ,T) be an RN-space.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to x in X if, for any $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that $\mu_{x_n-x}(\epsilon) > 1-\lambda$ whenever $n \geq N$.

(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for any $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that

$$\mu_{x_m-x_n}(\epsilon) > 1-\lambda$$

whenever $n \geq m \geq N$.

(3) An RN-space (X,μ,T) is said to be *complete* if every Cauchy sequence in X is convergent to a point in X.

3 Approximation of random C^*-ternary algebras homomorphisms

In this section, we approximate random C^*-ternary algebras homomorphisms of a Cauchy-Jensen additive mapping (see also [34–45]).

For a given mapping $f: A \to B$, we define

$$\begin{aligned} &C_\mu f(x_1,\ldots,x_p,y_1,\ldots,y_d) \\ &\quad := 2f\left(\frac{\sum_{j=1}^p \mu x_j}{2} + \sum_{j=1}^d \mu y_j\right) - \sum_{j=1}^p \mu f(x_j) - 2\sum_{j=1}^d \mu f(y_j) \end{aligned}$$

for all $\mu \in \mathbf{T}^1 := \{\lambda \in \mathbf{C} : |\lambda| = 1\}$ and $x_1,\ldots,x_p,y_1,\ldots,y_d \in A$.

One can easily show that a mapping $f: A \to B$ satisfies

$$C_\mu f(x_1,\ldots,x_p,y_1,\ldots,y_d) = 0$$

for all $\mu \in \mathbf{T}^1$ and $x_1,\ldots,x_p,y_1,\ldots,y_d \in A$ if and only if

$$f(\mu x + \lambda y) = \mu f(x) + \lambda f(y)$$

for all $\mu,\lambda \in \mathbf{T}^1$ and $x,y \in A$.

We use the following lemma in this paper.

Lemma 3.1 ([46]) *Let $f: A \to B$ be an additive mapping such that $f(\mu x) = \mu f(x)$ for all $x \in A$ and $\mu \in \mathbf{T}^1$. Then the mapping f is $\mathbf{C}$-linear.*

Theorem 3.2 *Let r, s, and θ be non-negative real numbers such that $0 < r \neq 1$, $0 < s \neq 3$. Let $\varphi : A^{p+d} \to D^+$ ($d \geq 2$) and $\psi : A^3 \to D^+$ such that*

$$\varphi_{a(x_1,\ldots,x_p,y_1,\ldots,y_d)}(t) = \varphi_{x_1,\ldots,x_p,y_1,\ldots,y_d}\left(\frac{t}{a^r}\right) \tag{1}$$

and

$$\psi_{a(x,y,z)}(t) = \psi_{x,y,z}\left(\frac{t}{a^s}\right) \tag{2}$$

for all $x_1,\ldots,x_p,y_1,\ldots,y_d,x,y,z \in A$ and $a \in \mathbf{C}$. Suppose that $f : A \to B$ is a mapping with $f(0) = 0$, satisfying

$$\mu_{C_\mu f(x_1,\ldots,x_p,y_1,\ldots,y_d)}(t) \geq \varphi_{x_1,\ldots,x_p,y_1,\ldots,y_d}(t) \tag{3}$$

and

$$\mu_{f([x,y,z])-[f(x),f(y),f(z)]}(t) \geq \psi_{x,y,z}(t) \tag{4}$$

for all $\mu \in \mathbf{T}^1$, $x_1,\ldots,x_p,y_1,\ldots,y_d,x,y,z \in A$, and $t > 0$. Then there exists a unique C^*-ternary algebra homomorphism $H : A \to B$ such that

$$\mu_{f(x)-H(x)}(t) \geq \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{x,\ldots,x}_{d}}\bigl(2t\bigl(d - d^r\bigr)\bigr) \tag{5}$$

for all $x \in A$ and $t > 0$.

Proof We prove the theorem when $0 < r < 1$ and $0 < s < 3$. Similarly, one can prove the theorem for other cases. Letting $\mu = 1$, $x_1 = \cdots = x_p = 0$, and $y_1 = \cdots = y_d = x$ in (3), we get

$$\mu_{2f(dx)-2df(x)}(t) \geq \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{x,\ldots,x}_{d}}(t) \tag{6}$$

for all $x \in A$ and $t > 0$. If we replace x by $d^n x$ in (6), we get

$$\mu_{\frac{1}{d^{n+1}}f(d^{n+1}x)-\frac{1}{d^n}f(d^n x)}(t) \geq \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{x,\ldots,x}_{d}}\bigl(2dtd^{(1-r)n}\bigr)$$

for all $x \in A$, all non-negative integers n and $t > 0$. Therefore,

$$\mu_{\frac{1}{d^{n+m}}f(d^{n+m}x)-\frac{1}{d^m}f(d^m x)}(t) \geq \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{x,\ldots,x}_{d}}\left(\frac{2dt}{\sum_{k=m}^{m+n} d^{(r-1)k}}\right) \tag{7}$$

for all $x \in A$, non-negative integers n, m and $t > 0$. From this, it follows that the sequence $\{\frac{1}{d^n}f(d^n x)\}$ is a Cauchy sequence for all $x \in A$. Since B is complete, the sequence $\{\frac{1}{d^n}f(d^n x)\}$ converges. Thus one can define the mapping $H : A \to B$ by

$$H(x) := \lim_{n\to\infty} \frac{1}{d^n} f\bigl(d^n x\bigr)$$

for all $x \in A$. Moreover, letting $m = 0$ and passing to the limit $n \to \infty$ in (7), we get (5). It follows from (3) that

$$\begin{aligned}
&\mu_{2H(\frac{\sum_{j=1}^{p}\mu x_j}{2}+\sum_{j=1}^{d}\mu y_j)-\sum_{j=1}^{p}\mu H(x_j)-2\sum_{j=1}^{d}\mu H(y_j)}(t) \\
&\quad = \lim_{n\to\infty} \mu_{\frac{1}{d^n}(2f(d^n\frac{\sum_{j=1}^{p}\mu x_j}{2}+d^n\sum_{j=1}^{d}\mu y_j)-\sum_{j=1}^{p}\mu f(d^n x_j)-2\sum_{j=1}^{d}\mu f(d^n y_j))}(t) \\
&\quad \geq \lim_{n\to\infty} \varphi_{d^n(x_1,\ldots,x_p,y_1,\ldots,y_d)}\bigl(d^n t\bigr) \\
&\quad \geq \lim_{n\to\infty} \varphi_{x_1,\ldots,x_p,y_1,\ldots,y_d}\left(\frac{d^n}{d^{nr}}t\right) \\
&\quad = 1
\end{aligned}$$

for all $\mu \in \mathbf{T}^1$, $x_1,\ldots,x_p,y_1,\ldots,y_d \in A$, and $t > 0$. Hence we have

$$2H\left(\frac{\sum_{j=1}^{p}\mu x_j}{2}+\sum_{j=1}^{d}\mu y_j\right) = \sum_{j=1}^{p}\mu H(x_j) + 2\sum_{j=1}^{d}\mu H(y_j)$$

for all $\mu \in \mathbf{T}^1$ and $x_1,\ldots,x_p,y_1,\ldots,y_d \in A$ and so

$$H(\lambda x + \mu y) = \lambda H(x) + \mu H(y)$$

for all $\lambda, \mu \in \mathbf{T}^1$ and $x, y \in A$. Therefore, by Lemma 3.1, the mapping $H : A \to B$ is $\mathbf{C}$-linear. It follows from (4) that

$$\begin{aligned}
&\mu_{H([x,y,z])-[H(x),H(y),H(z)]}(t) \\
&\quad = \lim_{n\to\infty} \mu_{\frac{1}{d^{3n}}(f([d^n x,d^n y,d^n z])-[f(d^n x),f(d^n y),f(d^n z)])}(t) \\
&\quad = \lim_{n\to\infty} \mu_{(f([d^n x,d^n y,d^n z])-[f(d^n x),f(d^n y),f(d^n z)])}\bigl(d^{3n} t\bigr) \\
&\quad \geq \lim_{n\to\infty} \psi_{d^n x,d^n y,d^n z}\bigl(d^{3n} t\bigr) \\
&\quad \geq \lim_{n\to\infty} \psi_{x,y,z}\left(\frac{d^{3n}}{d^{ns}}\right) = 1
\end{aligned}$$

for all $x, y, z \in A$ and $t > 0$ and so

$$H\bigl([x,y,z]\bigr) = \bigl[H(x),H(y),H(z)\bigr]$$

for all $x, y, z \in A$.

Now, let $T : A \to B$ be another generalized Cauchy-Jensen additive mapping satisfying (5). Then we have

$$\begin{aligned}
\mu_{H(x)-T(x)}(t) &= \lim_{n\to\infty} \mu_{\frac{1}{d^n}(f(d^n x)-T(d^n x))}(t) \\
&= \lim_{n\to\infty} \mu_{f(d^n x)-T(d^n x)}\bigl(d^n t\bigr) \\
&\geq \lim_{n\to\infty} \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{d^n x,\ldots,d^n x}_{d}}\bigl(2td^n\bigl(d-d^r\bigr)\bigr)
\end{aligned}$$

$$\begin{aligned}
&\geq \lim_{n\to\infty} \varphi_{\underbrace{0,\ldots,0}_{p},\underbrace{x,\ldots,x}_{d}}\left(\frac{2td^n(d-d^r)}{d^{nr}}\right) \\
&= 1
\end{aligned}$$

for all $x \in A$ and $t > 0$. So we can conclude that $H(x) = T(x)$ for all $x \in A$. This proves the uniqueness of H. Thus the mapping $H : A \to B$ is a unique C^*-ternary algebra homomorphism satisfying (5). This completes the proof. □

Theorem 3.3 *Let $r < 1$, $s < 2$, θ be non-negative real numbers and let $f : A \to B$ be a mapping satisfying* (1), (2), (3) *and* (4). *If there exist a real number $\lambda > 1$ $(0 < \lambda < 1)$ and an element $x_0 \in A$ such that*

$$\lim_{n\to\infty} \frac{1}{\lambda^n} f(\lambda^n x_0) = e' \qquad \left(\lim_{n\to\infty} \lambda^n f\left(\frac{x_0}{\lambda^n}\right) = e'\right),$$

then the mapping $f : A \to B$ is a C^*-ternary algebra homomorphism.

Proof By using the proof of Theorem 3.2, there exists a unique C^*-ternary algebra homomorphism $H : A \to B$ satisfying (5). Now,

$$H(x) = \lim_{n\to\infty} \frac{1}{\lambda^n} f(\lambda^n x) \qquad \left(H(x) = \lim_{n\to\infty} \lambda^n f\left(\frac{x}{\lambda^n}\right)\right) \tag{8}$$

for all $x \in A$ and all real numbers $\lambda > 1$ $(0 < \lambda < 1)$. Therefore, by the assumption, we get that $H(x_0) = e'$. Let $\lambda > 1$ and $\lim_{n\to\infty} \frac{1}{\lambda^n} f(\lambda^n x_0) = e'$. It follows from (4) and (8) that

$$\begin{aligned}
&\mu_{[H(x),H(y),H(z)]-[H(x),H(y),f(z)]}(t) \\
&\quad = \mu_{H[x,y,z]-[H(x),H(y),f(z)]}(t) \\
&\quad = \lim_{n\to\infty} \mu_{\frac{1}{\lambda^{2n}}(f([\lambda^n x,\lambda^n y,z])-[f(\lambda^n x),f(\lambda^n y),f(z)])}(t) \\
&\quad = \lim_{n\to\infty} \mu_{f([\lambda^n x,\lambda^n y,z])-[f(\lambda^n x),f(\lambda^n y),f(z)]}\left(\lambda^{2n} t\right) \\
&\quad \geq \lim_{n\to\infty} \psi_{\lambda^x,\lambda^y,\lambda^z}\left(\lambda^{2n} t\right) \\
&\quad = \psi_{x,y,z}\left(\frac{\lambda^{2n}}{\lambda^{2ns}} t\right) \\
&\quad = 1
\end{aligned}$$

for all $x \in A$ and $t > 0$ and so

$$\left[H(x), H(y), H(z)\right] = \left[H(x), H(y), f(z)\right]$$

for all $x, y, z \in A$. Letting $x = y = x_0$ in the last equality, we get $f(z) = H(z)$ for all $z \in A$.

Similarly, one can show that $H(x) = f(x)$ for all $x \in A$ when $0 < \lambda < 1$ and $\lim_{n\to\infty} \lambda^n f(\frac{x_0}{\lambda^n}) = e'$. Therefore, the mapping $f : A \to B$ is a C^*-ternary algebra homomorphism. This completes the proof. □

Theorem 3.4 *Let $r > 1$, $s > 3$, θ be non-negative real numbers and let $f : A \to B$ be a mapping satisfying* (3) *and* (4). *If there exists a real number $0 < \lambda < 1$ $(\lambda > 1)$ and an element*

$x_0 \in A$ such that

$$\lim_{n\to\infty} \frac{1}{\lambda^n} f(\lambda^n x_0) = e' \qquad \left(\lim_{n\to\infty} \lambda^n f\left(\frac{x_0}{\lambda^n}\right) = e'\right),$$

then the mapping $f: A \to B$ is a C^*-ternary algebra homomorphism.

Proof The proof is similar to the proof of Theorem 3.3 and we omit it. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

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